WALL COLLISION OF WAVES FROM ONE-DIMENSIONAL GAS DETONATIONS WITH LARGE AND NEGLIGIBLY SMALL IGNITION INDUCTION PERIODS

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The present problem is of interest in relation to the recently observed interaction of transverse discontinuities in a gas detonation front [1–4] and also other phenomena in wave gas dynamics, a new area of science concerned with wave interaction in supersonic flows [5]. A difference from earlier studies [6–8], and also those of Shchelkin [9] and Stanykovich [7, 10] is that I assume that the reflected wave is not a shock wave but a detonation one, which propagates in a shock-compressed but as yet unreacted explosive gas mixture, which is considered as an ideal gas. This is possible, for example, if the induction period for ignition in the incident wave greatly exceeds the induction period in the reflected detonation wave. For gas detonations the ratio of the initial pressure to the pressure behind the wave is not [11] negligibly small (p_0/p_1 of 1/6 to 1/20), and so the incident wave is taken as of arbitrary form.

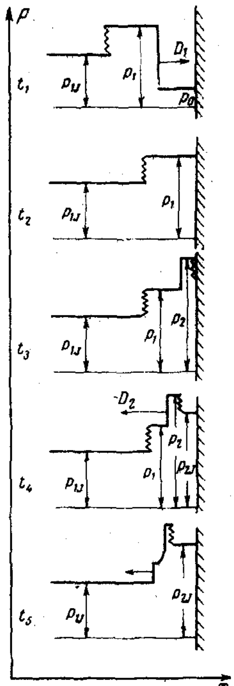

Fig. 1

Figure 1 illustrates the most characteristic stages of this collision and reflection for times t_1–t_5 in (p, x) coordinates. Subscripts 0, 1, 2 on this indicate the initial state and the parameters in the incident and reflected waves.

Considering the flow in coordinates linked to the wave front, and assuming that the gas at the wall is at rest, i.e., $u_0 = u_2 = 0$, we get as follows:

for the incident wave

$$\rho_0 D_1 = \rho_1 (D_1 - u_1), \qquad p_1 - p_0 = \rho_0 D_1 u_1, \qquad \frac{\rho_1}{\rho_0} = \frac{2\gamma - (\gamma - 1) P_1}{2\gamma - (\gamma + 1) P_1}; \tag{1}$$

for the reflected wave

$$\rho_2 D_2 = \rho_1 (D_2 + u_1), \qquad p_2 - p_1 = \rho_1 (D_2 + u_1) u_1,$$

$$\frac{\rho_2}{\rho_1} = \frac{2\gamma + (\gamma + 1) P_2}{2(\gamma + q) + (\gamma - 1) P_2},$$

$$P_1 = 1 - \frac{p_0}{p_1}, \qquad P_2 = \frac{p_2}{p_1} - 1, \tag{2}$$

$$q = \frac{Q(\gamma - 1)}{p_1/\rho_1} = \gamma(\gamma - 1)\frac{Q}{c_1^2}, \qquad \gamma = \frac{c_p}{c_v}. \tag{3}$$

Here P_1 and P_2 are the relative pressure differences in the incident and reflected waves respectively; D_1 and D_2 are the velocities of propagation of those waves; q is the dimensionless energy release (ratio of the heat of combustion of unit mass of the mixture to the initial internal energy of that mass); c_p and c_v are the specific heats; and c_1 is the velocity of sound in the shock-compressed gas behind the incident wave. The last is given by c_0 and the Mach number M_1 of the incident wave

$$c_1^2 = c_0^2\,\frac{(2\gamma M_1^2 - \gamma + 1)\,[2 + (\gamma - 1) M_1^2]}{(\gamma + 1)^2 M_1^2}. \tag{4}$$

From (1), with the momentum equation (2), we have

$$P_2 = P_1\left[\frac{2\gamma - (\gamma - 1) P_1}{2\gamma - (\gamma + 1) P_1}\left(\frac{D_2}{D_1} + 1\right) - 1\right]. \tag{5}$$

Also, from (1) and (2) we have, respectively, for the square of the relative velocity of the gas

$$u_1^2 = (p_1 - p_0)\left(\frac{1}{\rho_0} - \frac{1}{\rho_1}\right), \qquad u_1^2 = (p_2 - p_1)\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right). \tag{6}$$

From (6) we have from equations (1) and (2) that

$$\frac{P_2 (P_2 - q)}{\gamma + {}^1/_2 P_2 (\gamma + 1)} = \frac{P_1^2}{\gamma - {}^1/_2 P_1 (\gamma + 1)}. \tag{7}$$

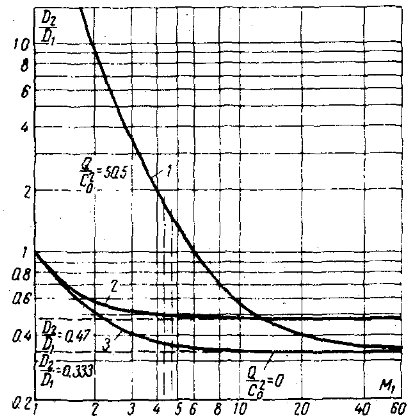

Fig. 2

We denote the right side of this by F to get that

$$P_2^2 - \frac{(\gamma + 1) F + 2q}{2} P_2 - \gamma F = 0. \tag{8}$$

Then

$$P_2 = \frac{(\gamma + 1) F + 2q \pm \sqrt{[(\gamma + 1) F + 2q]^2 + 16\gamma F}}{4}. \tag{9}$$

The positive sign is taken for the root in this formula, since detonation must correspond to a pressure change such that $P_2 > 0$. For Q = 0 formula (9) reduces to the standard formula [6, 7] for the pressure in a shock wave reflected from an absolutely rigid wall

$$P_2 = \frac{\gamma P_1}{\gamma - {}^1/_2 P_1 (\gamma - 1)}. \tag{10}$$

We equate the right sides of (5) and (9) to get

$$\frac{D_2}{D_1} = \frac{(\gamma - 3)F + 2q + \sqrt{[(\gamma + 1)F + 2q]^2 + 16\gamma F}}{4(F + P_1)}. \quad (11)$$

One of the basic assumptions here is that ignition is completely absent in the gas in state 1 for certain time, so this treatment must be considered as a limit to real processes of pulsating detonation, since the gas in state 1 may consist not only of the shock-compressed initial mixture but also of detonation products, due (for example) to the periodic structure of the wave or to the presence of a fine structure of interacting discontinuities [1, 12, 13]. The iso-entropic relation of pressure to density [10, 14] applies to the detonation products

$$p\rho^{-\gamma} = \text{const}. \quad (12)$$

In this connection it is of interest to consider another limiting case, in which we assume that the entire region of compressed gas in the detonation wave behind the leading edge incident on the wall is filled by detonation products. Stanyukovich [7, 10] discussed this case in 1946 for a strong detonation wave, i.e., on the assumption $p_1 \gg p_0$. Here this problem is solved afresh for an arbitrary wave (i.e., not assuming p_0 small relative to p_1). We put Q = 0 in (2), while (1) is replaced by the result derived from (12) to give the expression for the density ratio as

$$\rho_1 / \rho_0 = (\gamma + P_1) / \gamma. \quad (13)$$

Then (1) is solved with (13) and (2) modified by putting Q = 0 to get the following formulas for P_2 and D_2/D_1:

$$P_2 = P_1 \frac{(\gamma + 1)P_1 + \sqrt{(\gamma + 1)^2 P_1^2 + 16\gamma^2}}{4\gamma}, \quad (14)$$

$$\frac{D_2}{D_1} = \frac{(\gamma - 3)P_1 + \sqrt{(\gamma + 1)^2 P_1^2 + 16\gamma^2}}{4(\gamma + P_1)}. \quad (15)$$

Calculation of P_1 from the known M_1 of the incident wave for substitution in (9)–(11) is performed as for a shock wave without energy release:

$$P_{1s} = 1 - \frac{p_0}{p_{1s}} = 1 - \frac{\gamma + 1}{2\gamma M_1^2 - (\gamma - 1)}. \quad (16)$$

It is convenient to deduce P_{1s} from the following formula in determining D_2/D_1 as a function of integer values of $M_1 \geq 2$:

$$(P_{1s})_M = \Big(\gamma \sum_{N=2}^{N=M} 2(2N-1)\Big)\Big[1 + \gamma\Big(1 + \sum_{N=2}^{N=M} 2(2N-1)\Big)\Big]^{-1}. \quad (17)$$

The P_1 for substitution into (14) and (15) is calculated as for a detonation wave via the formula

$$P_{1J} = 1 - \frac{p_0}{p_{1J}} = 1 - \frac{\gamma + 1}{\gamma M_1^2 + 1}. \quad (18)$$

Figure 2 shows D_2/D_1 as a function of M_1 for a hydrogen-oxygen mixture with γ = 1.4 for these two limiting cases (curves 1 and 2); for comparison, I give curve 3 calculated from (11) subject to $Q/c_0^2 = 0$. The broken lines are the asymptotes D_2/D_1 = 0.333 and D_2/D_1 = 0.47, which correspond to the solutions for strong shock and detonation waves. As regards the asymptotic solutions we may note that the results for M_1 of 4–7 (the range characteristic of wave propagation in gases) show that neglect of p_0 relative to p_1 leads to errors of 12 and 6% respectively for the collision of shock and detonation waves with a wall.

The two vertical broken lines in Fig. 2 delimit the region of limiting M_1 for incident detonation waves; to the left of this region lie the D_2/D_1 obtained by reflection of shock waves formed ahead of the flame under predetonation conditions, in the so-called unstable detonations [15]. To the right of this region as far as the point of intersection of curves 1 and 2 (at M_1 = 13) lies the range of D_2/D_1 for pulsating detonation. A reflected detonation wave cannot occur to the right of this point, since for a mixture with this Q/c_0^2 there cannot be incident detonation waves with these large M_1. For instance, the calculated maximum velocity of an incident detonation wave for a stoichiometric hydrogen-oxygen mixture is 5100 m/sec, which corresponds to M_1 = 10, while the actually recorded detonation speeds do not exceed Schmidt's [16] value D_1 = 4440 m/sec (M_1 = 8.65) for p_0 = 800 kg/cm^2. Hence all the experimental D_2/D_1 must lie within the acute-angled sector formed by curves 1 and 2 to the left of their point of intersection.

Curve 1 tends to infinity near M_1 = 1, which physically merely implies simultaneous spontaneous ignition of the slightly perturbed gas at all points. Here (9) shows that P_2 tends to a constant equal to $q = \gamma(\gamma - 1)Q/c_0^2$.

I am indebted to Ya. K. Troshin, who proposed this study of the collision of detonation waves.

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